Lecture 4: corner and blob detection

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Recap of last lecture: 2D Edge detection

Canny

• compute gradient: $\nabla S(x, y)$

• compute gradient directions: $\hat{n} = \frac{\nabla S(x, y)}{|\nabla S(x, y)|}$

• find local maxima of gradient magnitude $|\nabla S(x, y)|$ in direction $\hat{n}$
Recap of last lecture: 2D Edge detection

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equivalent to finding $\frac{d^2}{dr^2}S(x, y) = 0$ where $r$ is distance in direction $\hat{n}$
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Marr-Hildreth:

- find $\nabla^2 S(x, y) = \left( \frac{d^2}{dr^2} + \frac{d^2}{dq^2} \right) S(x, y) = 0$

  where $r$ is in direction of edge and $q$ is perpendicular
Recap of last lecture: 2D Edge detection

**Canny**

- compute gradient: \( \nabla S(x, y) \)

- compute gradient directions: \( \hat{n} = \frac{\nabla S(x, y)}{|\nabla S(x, y)|} \)

- find local maxima of gradient magnitude \( |\nabla S(x, y)| \) in direction \( \hat{n} \)

Equivalent to finding \( \frac{d^2}{dr^2}S(x, y) = 0 \) where \( r \) is distance in direction \( \hat{n} \)

**Marr-Hildreth:**

- find \( \nabla^2 S(x, y) = \left( \frac{d^2}{dr^2} + \frac{d^2}{dq^2} \right) S(x, y) = 0 \)

- where \( r \) is in direction of edge and \( q \) is perpendicular

Additional \( \frac{d^2}{dq^2}S(x, y) \) term introduces noise
Recap of last lecture: 2D Edge detection

**Canny:** slower, more reliable

- compute gradient: \( \nabla S(x, y) \)

- compute gradient directions: \( \hat{n} = \frac{\nabla S(x, y)}{|\nabla S(x, y)|} \)

- find local maxima of gradient magnitude \( |\nabla S(x, y)| \) in direction \( \hat{n} \)

equivalent to finding \( \frac{d^2}{dr^2} S(x, y) = 0 \) where \( r \) is distance in direction \( \hat{n} \)

**Marr-Hildreth:** fast, noisy

- find \( \nabla^2 S(x, y) = \left( \frac{d^2}{dr^2} + \frac{d^2}{dq^2} \right) S(x, y) = 0 \)

- where \( r \) is in direction of edge and \( q \) is perpendicular

additional \( \frac{d^2}{dq^2} S(x, y) \) term introduces noise
Recap of last lecture: 2D Edge detection

- accelerated edge detection:
  - truncated tails of filter
  - leveraged separability of filter

- discovered we need more than edges: aperture problem

  $\rightarrow$ corners
Correlation

Sums are just over elements in the patch

\[ c(x, y) = \frac{\sum_u \sum_v P(u, v)I(x + u, y + v)}{\sqrt{\sum_u \sum_v P(u, v)^2 \sum_{u'} \sum_{v'} I(x + u', y + v')^2}} \]
Correlation

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\[ P(u, v) \]

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\[
c(x, y) = \frac{p^T I}{|p||I|}
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\[
c(x, y) = \frac{p^T I}{|p||I|} = \cos(\theta)
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\[ c(x, y) = \frac{p^T I}{|p||I|} = \cos(\theta) \]

\[ I' = \alpha I + \beta \quad \text{brightness} \]
Correlation

Sums are just over elements in the patch:

\[ c(x, y) = \frac{\sum_u \sum_v P(u, v)I(x + u, y + v)}{\sqrt{\sum_u \sum_v P(u, v)^2 \sum_{u'} \sum_{v'} I(x + u', y + v')^2}} \]

Alters length of vector, not angle.

\[ c(x, y) = \frac{p^T I}{|p||I|} = \cos(\theta) \]

Contrast: multiplicative, alters length of vector, not angle.

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brightness
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\[I' = \alpha I + \beta \]

Brightness alters angle

Contrast: multiplicative
Alters length of vector, not angle
Correlation: remove mean

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c(x, y) = \frac{p^T I}{|p||I|} = \cos(\theta)
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\[
\tilde{I}(x, y) = I(x, y) - \langle I(x, y) \rangle
\]

Brightness: alters angle
Contrast: multiplicative
  alters length of vector, not angle

\[
I' = \alpha I + \beta \rightarrow \text{brightness}
\]
Correlation: cross-correlation

\[ \tilde{c}(x, y) = \frac{\sum_u \sum_v \tilde{P}(u, v) \tilde{I}(x + u, y + v)}{\sqrt{\sum_u \sum_v \tilde{P}(u, v)^2 \sum_{u'} \sum_{v'} \tilde{I}(x + u', y + v')^2}} \]

\[ \tilde{c}(x, y) = \frac{\tilde{p}^\top \tilde{I}}{||\tilde{p}|| \tilde{I}} = \cos(\theta) \]

brightness: alters angle

\[ \tilde{I}(x, y) = I(x, y) - \langle I(x, y) \rangle \]

contrast: multiplicative
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brightness
A signature of corners

\[ c(x, y) \]
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Summary

Correlating with

- **0D patch**: flat
- **1D patch** (edge): falls off quickly in one direction, constant in other
- **2D patch** (corners): peak at intersection, falls off quickly in all directions
- textural patches also result in peaks
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How can we use this fact for edge detection?
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**How can we use this fact for edge detection?**

- **match with template corners**: computationally expensive
- find points where **auto-correlation** falls off quickly in all directions
Corners as interest points

\[ M = \sum w(x, y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix} \]

2 x 2 matrix of image derivatives (averaged in neighborhood of a point).

Notation:

\[ I_x \leftrightarrow \frac{\partial I}{\partial x} \]
\[ I_y \leftrightarrow \frac{\partial I}{\partial y} \]
\[ I_x I_y \leftrightarrow \frac{\partial I}{\partial x} \frac{\partial I}{\partial y} \]
Looking more closely at the matrix

First, consider an axis-aligned corner:

\[ M = \sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \]

This means dominant gradient directions align with x or y axis.

Look for locations where both \( \lambda \)'s are large.

If either \( \lambda \) is close to 0, then this is not corner-like.

What if we have a corner that is not aligned with the image axes?
Looking more closely at the matrix

Since $M$ is symmetric, we have

$$M = X \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} X^T$$

$$Mx_i = \lambda_i x_i$$

The *eigenvalues* of $M$ reveal the amount of intensity change in the two principal orthogonal gradient directions in the window.
Responses to edges corners and flat regions

“edge”:
\[ \lambda_1 >> \lambda_2 \]
\[ \lambda_2 >> \lambda_1 \]

“corner”:
\[ \lambda_1 \text{ and } \lambda_2 \text{ are large,} \]
\[ \lambda_1 \sim \lambda_2; \]

“flat” region
\[ \lambda_1 \text{ and } \lambda_2 \text{ are small;} \]

\[ \text{cornerness}(x, y) = \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2 \]
What type of applications are corners useful for?

- Autostitch: photo-mosaic
- Matching objects taken with different cameras/viewpoints
What type of applications are corners useful for?

Autostitch: photo-mosaic
- works well

Matching objects taken with different cameras/viewpoints
- fails
Properties of Harris corners

Harris corners are rotationally invariant  
**But are they scale invariant?**
Properties of Harris corners

Harris corners are rotationally invariant
But are they scale invariant?

All regions classified as edges
Properties of Harris corners

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Corner
Properties of Harris corners: need scale invariant features

Harris corners are rotationally invariant
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All regions classified as edges

Corner